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$\therefore AMND = MBCN.$

Note. If the angles at C and D are right, MN, of course, is perpendicular to CD. As the proposer states this theorem, it is too restricted.

Also solved by G. B. M. Zerr, J, A. Caparo, A. H. Holmes, and analytically by V. M. Spunar.

341. Proposed by FRANK LOXLEY GRIFFIN, S. M., Ph. D., Instructor in Mathematics, Williams College.

Given $\rho = \cos(m/n)\theta$, where m and n are integers without a common factor. Deduce rules for finding by inspection:

(1) The angle between the beginning and end of any loop of this curve; (2) The number of distinct loops. [A loop is a portion of the curve between consecutive zero radii vectores.]

Solution by the PROPOSER.

(1) Consecutive values of $\frac{m}{n}\theta$ which make ρ vanish must differ by π. Hence, consecutive values of θ differ by $n\pi/m$.

(2) Since ρ is a periodic function of θ, the period being $\frac{2n\pi}{m}$, the rectangular coordinates have the period $2n\pi$. The whole figure being repeated with that period, attention may be confined to the interval $\theta = \frac{n}{m}\frac{\pi}{2}$ to $\theta = \frac{n}{m}\frac{\pi}{2} + 2n\pi$. In this interval there are $2m$ loops, as follows from (1). It remains to be seen when these loops will be all distinct.

The coincidence of two loops occurs only when a loop in which ρ is negative repeats one in which ρ is positive. In that case, any point (ρ_1, θ_1) on one loop coincides with $[-\rho_1, \theta_1 + \pi(2\lambda+1)]$ on the other [λ, some integer]. But, since $-\rho_1 = a\cos\frac{m}{n}[\theta_1 + \pi(2\lambda+1)]$, and $-\rho_1 = -a\cos\frac{m}{n}\theta_1$, it is clear that $\frac{m}{n}(2\lambda+1)\pi$ is an odd multiple of π, say $\frac{m}{n}(2\lambda+1) = 2k+1$. This is impossible if either m or n is even. Moreover, if both are odd, λ and k can always be found to satisfy the condition; *e. g.*, $\lambda = \frac{n-1}{2}$, $k = \frac{m-1}{2}$. Each loop for $\rho < 0$ will then repeat a loop for $\rho > 0$. Hence the number of distinct loops is (a) $2m$ if either m or n is even, (b) m if both m and n are odd.

Remark. The curve $\rho = \sin\frac{m}{n}\theta$ is simply the foregoing curve rotated through the angle $-\frac{n}{m}\frac{\pi}{2}$. Hence the conclusions reached above apply equally to this curve.

Also solved by G. B. M. Zerr, V. M. Spunar, nad J. W. Clawson, a Sophomore in Williams College.